

Do supersymmetric anti-de Sitter black rings exist?

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ABSTRACT: We determine the most general near-horizon geometry of a supersymmetric, asymptotically anti-de Sitter, black hole solution of five-dimensional minimal gauged supergravity that admits two rotational symmetries. The near-horizon geometry is that of the supersymmetric, topologically spherical, black hole solution of Chong et al. This proves that regular supersymmetric anti-de Sitter black rings with two rotational symmetries do not exist in minimal supergravity. However, we do find a solution corresponding to the near-horizon geometry of a supersymmetric black ring held in equilibrium by a conical singularity, which suggests that nonsupersymmetric anti-de Sitter black rings may exist but cannot be "balanced" in the supersymmetric limit.

KEYWORDS: Black Holes in String Theory, Black Holes, Supergravity Models.

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1. Introduction

Supersymmetric black hole solutions of five-dimensional gauged supergravity have been known for a few years [1–5]. According to the AdS/CFT correspondence [6], these black holes should correspond to 1/16 BPS states of $\mathcal{N} = 4$ $SU(N)$ super Yang-Mills theory on $R \times S^3$ or, equivalently, gauge-invariant local operators of this theory on R^4 . A puzzle arises [7] because BPS operators carry five independent conserved charges: their spins J_1, J_2 and R-charges Q_1, Q_2, Q_3 (where (Q_1, Q_2, Q_3) is a weight vector of the $SU(4)$ R-symmetry group), whereas the most general known asymptotically $AdS_5 \times S^5$ supersymmetric black hole solution has only 4 independent conserved charges [5].

There are two ways that this puzzle could be resolved. (i) We already know the most general supersymmetric black hole solution. Although generic 1/16-BPS operators have 5 independent charges, only a subset of these operators gives a large enough entropy to correspond to a macroscopic event horizon and this subset has only 4 independent charges. (Maybe because of finite coupling effects in the CFT.) This possibility has been explored in [8]. (ii) There exists a 5-parameter supersymmetric black hole solution that remains to be discovered.

If (ii) is correct, there appears to be some tension with what is known about black holes in *minimal* gauged supergravity. One can truncate to this theory by setting $Q_1 = Q_2 =$

$Q_3 = Q$. This theory admits a 4-parameter non-supersymmetric black hole solution [3]. The 4 parameters are the 4 conserved charges of this theory, namely J_1, J_2, Q and the mass M . One might expect this to be the most general black hole of spherical topology. In the supersymmetric limit, one loses two parameters: supersymmetric black holes are parameterized by J_1 and J_2 . Returning to general (unequal) Q_i , we would expect the most general nonsupersymmetric black hole to be parameterized by the 6 conserved charges $J_1, J_2, Q_1, Q_2, Q_3, M$ and losing 2 parameters in the supersymmetric limit would take one to the 4-parameter solution of [5]. There does not seem to be any room for an additional parameter.

This objection rests on the assumption that black holes should be characterized by their conserved charges. Even in four dimensions, there is no uniqueness theorem for asymptotically AdS black holes, so maybe this assumption is incorrect even for topologically spherical AdS black holes. Furthermore, we know that this assumption is violated by black rings [9] in five asymptotically flat dimensions, which can require nonconserved charges to specify them fully [10]. It is natural to guess that the same is true in AdS. So perhaps the black holes of (ii) are supersymmetric AdS black rings.¹

The goal of this paper is to classify supersymmetric black holes in five-dimensional gauged supergravity. Unfortunately, finding rotating black hole solutions is hard, even with supersymmetry. We shall therefore adopt the approach initiated in [11] of classifying *near-horizon geometries* of supersymmetric black holes.² Obviously the existence of a near-horizon geometry with certain properties cannot be taken as a proof of the existence of a full black hole solution with those properties but this approach *can* be used to rule out certain types of solution. For example, if we find that near-horizon geometries with horizon topology $S^1 \times S^2$ are not possible then that would exclude supersymmetric black rings.

In minimal five-dimensional gauged supergravity, a classification of near-horizon geometries was attempted in [1]. However, the resulting equations proved too difficult to solve in full generality without additional assumptions. (This is in contrast with the ungauged theory, for which a full classification is possible [11].) The assumptions made in [1] were too restrictive to encompass the solutions of [3]. In this paper, we return to the equations of [1] and solve them by making a weaker assumption, satisfied by all known five-dimensional black hole solutions, whether asymptotically AdS or asymptotically flat, including black rings [9, 10, 15]. Our assumption is the existence of two rotational symmetries.³

Our result is that a supersymmetric asymptotically AdS black hole satisfying this assumption must have a near-horizon geometry locally isometric to that of the known supersymmetric black hole solution of [3]. Hence either supersymmetric AdS black rings do not exist, or they are rather unusual in that they do not admit two rotational symmetries,

¹Other possibilities are that the black holes of (ii) do not admit a five-dimensional interpretation or that they involve non-abelian gauge fields.

²See [12–14] for other recent work on the near-horizon geometry of supersymmetric AdS black holes.

³Note that the "stationary implies axisymmetric" theorem [16] for black holes only guarantees the existence of a single rotational symmetry. Furthermore it does not apply to supersymmetric black holes. However, one might expect the conclusion to be valid for such solutions since one can obtain them as limits of non-supersymmetric black holes.

which would mean that they have less symmetry than any known five-dimensional black hole solution.

Could there exist new supersymmetric black holes of spherical topology, e.g. a 3-parameter generalization of the solution of [3]? We noted above that this seems unlikely but we cannot exclude the possibility that there exists such a solution with the same near-horizon geometry as the 2-parameter solution of [3], i.e., that a parameter is lost in the near-horizon limit (although we are not aware of any example in which this happens).

Our analysis is local: we do not enforce spatial compactness of the horizon until the end. This enables us to demonstrate why supersymmetric AdS black rings with the same symmetries as asymptotically flat ones do not exist. We do indeed obtain a solution that corresponds to the near-horizon geometry of a black hole with $S^1 \times S^2$ topology. It is a warped product of AdS_3 and S^2 . However, the S^2 always has a conical singularity. This suggests that AdS black rings may exist, but they cannot be "balanced" in the supersymmetric limit without the presence of external forces corresponding to the conical singularity. The conical singularity vanishes when one takes the limit of vanishing cosmological constant, and one recovers the $AdS_3 \times S^2$ geometry of an asymptotically flat supersymmetric black ring [15].

Interestingly, when lifted to a solution of type IIB supergravity, our singular black ring near-horizon geometry is *locally* isometric to a solution obtained in [17]. It was shown in [17] that this solution *can* be extended to a globally regular metric in ten dimensions, but it does not appear possible to reduce the resulting solution to five dimension so any interpretation in terms of five dimensional black holes is lost.

This paper is organized as follows. We start by deriving a general constraint on black holes with two rotational symmetries. Then we review the results of [1], presenting the equations relevant to our analysis and deriving some general results. We solve these equations subject to the assumption of two rotational symmetries. There are only two interesting solutions and we show that these correspond to the near-horizon geometry of a supersymmetric black ring with a conical singularity and the near-horizon geometry of the solution of [3] respectively. Section 3 concludes. Some details of the analysis are relegated to appendices.

2. Supersymmetric near-horizon geometries

2.1 General constraints

The bosonic sector of minimal $D = 5$ gauged supergravity is Einstein-Maxwell theory with a negative cosmological constant and a Chern-Simons term for the Maxwell field \mathcal{F} . The unique maximally supersymmetric solution of this theory is AdS_5 with vanishing gauge field [18]. We shall denote by ℓ the radius of this AdS_5 solution.

Consider an asymptotically AdS_5 solution of this theory (not necessarily supersymmetric) that is stationary and admits two rotational symmetries, i.e., there is a $R \times U(1) \times U(1)$ isometry group. Let k, m_1, m_2 denote the Killing fields that generate time translations and rotations respectively. We assume that these Killing fields commute and also leave the

Maxwell field invariant. Now we can use a standard argument from the theory of stationary axisymmetric solutions [19]: the Bianchi identity and the fact that the Lie derivatives of \mathcal{F} along the Killing fields vanish imply that $m_1^\mu m_2^\nu \mathcal{F}_{\mu\nu}$ is constant. However, since the solution is asymptotically AdS_5 , we can find coordinates so that the asymptotic metric is

$$ds^2 \sim - \left(1 + \frac{r^2}{\ell^2}\right) dt^2 + \left(1 + \frac{r^2}{\ell^2}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi_1^2 + \cos^2 \theta d\phi_2^2), \quad (2.1)$$

where $k = \partial/\partial t$, $m_i = \partial/\partial\phi_i$ and $0 \leq \theta \leq \pi/2$. Therefore, in the asymptotic region, m_1 and m_2 vanish at $\theta = 0, \pi/2$ respectively. It follows that

$$m_1^\mu m_2^\nu \mathcal{F}_{\mu\nu} \equiv 0. \quad (2.2)$$

The same argument shows that $k^\mu m_i^\nu \mathcal{F}_{\mu\nu} \equiv 0$ although we shall not need this result.

2.2 Near-horizon limit

Given a supersymmetric black hole we can take a near-horizon limit as explained in [11] to obtain a supersymmetric near-horizon solution. We want to classify such solutions. Necessary and sufficient conditions for a near-horizon geometry to be a supersymmetric solution of this theory were worked out in [1] using the method introduced in [11]. One can introduce Gaussian null coordinates on the horizon so that the near-horizon metric is [11]

$$ds^2 = -r^2 \Delta(x)^2 dv^2 + 2dvdr + 2rh_a(x)dvdx^a + g_{ab}(x)dx^a dx^b, \quad (2.3)$$

where the horizon is at $r = 0$, $\partial/\partial v$ is Killing, Δ is non-negative, and the metric g_{ab} is the metric on a spatial cross-section of the horizon. We shall denote this 3-manifold as H . Supersymmetry implies the following [1]:

- There exists a globally defined unit 1-form Z on H :

$$g^{ab} Z_a Z_b = 1, \quad (2.4)$$

where g^{ab} is the inverse of g_{ab} .

- The (near-horizon) Maxwell field is

$$\mathcal{F} = \frac{\sqrt{3}}{2} \left(-dv \wedge d(r\Delta) - \star h - \frac{2}{\ell} \star Z \right), \quad (2.5)$$

where \star is the Hodge dual on H . Hence the Bianchi identity is

$$d \star \left(h + \frac{2}{\ell} Z \right) = 0. \quad (2.6)$$

- The following equations hold on H

$$\star dh - d\Delta - \Delta h = \frac{6\Delta}{\ell} Z, \quad (2.7)$$

$$\nabla_a Z_b = -\frac{\Delta}{2} (\star Z)_{ab} + g_{ab} \left(h \cdot Z + \frac{3}{\ell} \right) - Z_a h_b - \frac{3}{\ell} Z_a Z_b, \quad (2.8)$$

where $h \cdot Z \equiv g^{ab} h_a Z_b$. This implies that

$$dZ = -\Delta \star Z + h \wedge Z. \quad (2.9)$$

- The Ricci tensor of H is

$$R_{ab} = \left(\frac{\Delta^2}{2} + h \cdot h + \frac{4}{\ell} h \cdot Z \right) g_{ab} - h_a h_b - \nabla_{(a} h_{b)} - \frac{6}{\ell} h_{(a} Z_{b)} - \frac{6}{\ell^2} Z_a Z_b. \quad (2.10)$$

These equations are not the complete set derived in [1] but they are the only ones we will need here. We have not been able to solve these equations in full generality. However, we have obtained some general results that is it convenient to record here.

The near-horizon geometry is *static* if, and only if, the Killing field $V \equiv \partial/\partial v$ is hypersurface orthogonal, i.e., $V \wedge dV \equiv 0$. The following lemma gives the conditions for this to occur:

Lemma 1. The following conditions are equivalent: (a) the near-horizon geometry is static, (b) $dh \equiv 0$, (c) $\Delta \equiv 0$.

Proof. Assume (a). Then the rab components of $V \wedge dV \equiv 0$ give $dh \equiv 0$ so (a) implies (b).

Now assume (b). If Δ is nonzero then equation (2.7) implies that $Z = -(\ell/6)(h + d\Delta/\Delta)$ and hence Z is closed. Equation (2.9) then implies that $\Delta Z \wedge \star Z = 0$, hence $Z = 0$, in contradiction with the fact that Z has unit norm. Hence we must have $\Delta = 0$ everywhere. Hence (b) implies (c).

Finally assume (c). Equation (2.7) shows that $dh \equiv 0$. But $\Delta \equiv 0$ and $dh \equiv 0$ implies $V \wedge dV \equiv 0$. Hence (c) implies (a).

Our second lemma shows that Δ cannot vanish anywhere if the near-horizon geometry is non-static.

Lemma 2. If Δ vanishes at a point then Δ vanishes everywhere.

Proof. Take the divergence of (2.7) and use equations (2.6) and (2.8) to obtain

$$\nabla^2 \Delta = - \left(h + \frac{6}{\ell} Z \right) \cdot \nabla \Delta - \frac{8}{\ell} \left(h \cdot Z + \frac{3}{\ell} \right) \Delta. \quad (2.11)$$

Assume $\Delta = 0$ at p . Then, since $\Delta \geq 0$, Δ is at a (global) minimum so $d\Delta = 0$ at p . It then follows from this equation that $\nabla^2 \Delta = 0$ at p . However, since this is a minimum, the eigenvalues of the Hessian of Δ must be non-negative, and $\nabla^2 \Delta$ is the sum of the eigenvalues so they must all vanish. Hence the Hessian vanishes at p : $\partial_m \partial_n \Delta = 0$ at p .

Assume inductively that Δ and its first $2n$ derivatives vanish at p . Then the Taylor expansion at p begins

$$\Delta = \frac{1}{(2n+1)!} x^{i_1} \dots x^{i_{2n+1}} \partial_{i_1} \dots \partial_{i_{2n+1}} \Delta(p) + \dots, \quad (2.12)$$

where x^i are normal coordinates at p . Now Δ has to be non-negative but this leading term changes sign under $x \rightarrow -x$ and so we must require it to vanish. Therefore the $(2n+1)$ th derivatives of Δ must vanish at p . Now the expansion of Δ is

$$\Delta = \frac{1}{(2n+2)!} x^{i_1} \dots x^{i_{2n+2}} M_{i_1 \dots i_{2n+2}} + \dots, \quad (2.13)$$

where the symmetric tensor M is the $(2n + 2)$ th derivative of Δ . From our induction hypothesis and equation (2.11), we can obtain

$$\partial_{i_1} \dots \partial_{i_{2n}} \nabla^2 \Delta(p) = 0 \tag{2.14}$$

which implies

$$M_{i_1 \dots i_{2n} j j} = 0. \tag{2.15}$$

This implies that the leading term in the expansion of Δ is harmonic in R^3 . But we also need this term to be non-negative, which implies that it attains its minimum at the origin. Then from the maximum principle, it follows that this term must vanish everywhere, i.e., $M = 0$. So we've shown that the first $(2n + 2)$ derivatives of Δ vanish. It follows by induction, if $\Delta = 0$ at p then all derivatives of Δ vanish at p hence $\Delta \equiv 0$ by analyticity.⁴

In summary, these lemmas reveal that a static near-horizon geometry has $\Delta \equiv 0$ and a non-static one has $\Delta > 0$. We end this section by noting that a static near-horizon geometry can arise from a non-static black hole. Indeed, this is what happens for supersymmetric black rings in ungauged supergravity [15]. We shall see that the same appears to be true in gauged supergravity.

2.3 Including the symmetries

Consider a supersymmetric, asymptotically anti-de Sitter black hole admitting two rotational Killing fields m_1 and m_2 . The near-horizon solution will inherit these symmetries. Hence we are interested in classifying near-horizon solutions for which there exist two commuting Killing vector fields m_1, m_2 on H that preserve h, Δ and the Maxwell field \mathcal{F} . It follows from (2.5) that the Killing fields must also preserve Z . Furthermore, the near-horizon solution will also inherit the condition (2.2).

We can choose local coordinates $x^a = (\rho, x^i)$ so that $\partial/\partial x^i$ are Killing, the metric on H is

$$g_{ab} dx^a dx^b = d\rho^2 + \gamma_{ij}(\rho) dx^i dx^j, \tag{2.16}$$

and Δ and the components of h are functions only of ρ . It is convenient to allow the Killing fields $\partial/\partial x^i$ to be arbitrary linear combinations of m_1 and m_2 , i.e., $\partial/\partial x^i$ need not have closed orbits. We are then free to perform $GL(2, R)$ transformations on the coordinates x^i to simplify our analysis. We shall enforce the fact that orbits of m_1 and m_2 must close once we have determined a local solution.

It is convenient to parameterize the components of h as

$$h_i = \Gamma^{-1} \gamma_{ij} k^j, \quad h_\rho = -\frac{\Gamma'}{\Gamma}, \tag{2.17}$$

where $\Gamma(\rho)$ is positive and a prime denotes a derivative with respect to ρ .

The spatial components of the Maxwell field strength can be decomposed as

$$\mathcal{F}_{ab} dx^a \wedge dx^b = \frac{\sqrt{3}}{2} B_i(\rho) d\rho \wedge dx^i, \tag{2.18}$$

⁴One might object to the assumption of analyticity. However, none of the results in this paper will rely on this lemma.

where we have used equation (2.2) to deduce that the ij components must vanish. Comparing with equation (2.5) gives

$$Z = \frac{\ell}{2} (\star_2 B - h), \quad (2.19)$$

where \star_2 denotes the Hodge dual with respect to the two-dimensional metric γ_{ij} (with volume form η_2 oriented so that $d\rho \wedge \eta_2$ is the volume form of H).

The ρi component of equation (2.10) gives

$$0 = R_{\rho i} = -\frac{1}{2} \Gamma^{-1} \gamma_{ij} (k^j)', \quad (2.20)$$

hence we have

$$k^i = \text{constant}. \quad (2.21)$$

Substituting the expression (2.19) for Z into equation (2.7) gives

$$\Delta' + \frac{2\Delta\Gamma'}{\Gamma} = 0, \quad (2.22)$$

$$(\Gamma^{-1}k)' + 2\Delta \star_2 (\Gamma^{-1}k) = -3\Delta B, \quad (2.23)$$

where, for a 1-form $\omega_i(\rho)dx^i$, we define $\omega' = \omega'_i dx^i$. Equation (2.9) gives

$$\frac{\Delta\Gamma'}{\Gamma} \star_2 1 - \Gamma^{-1}k \wedge \star_2 B = 0, \quad (2.24)$$

$$(\star_2 B - \Gamma^{-1}k)' = \Delta B + \Delta \star_2 (\Gamma^{-1}k) - \frac{\Gamma'}{\Gamma} \star_2 B. \quad (2.25)$$

Solving equation (2.22) gives

$$\Delta = \frac{\Delta_0}{\Gamma^2}, \quad (2.26)$$

where Δ_0 is a non-negative constant. There are two cases to consider depending on whether $\Delta_0 > 0$ or $\Delta_0 = 0$. These correspond to non-static and static near-horizon geometries respectively.

2.4 Non-static near-horizon geometry

Consider first the non-static case $\Delta_0 > 0$. In this case, the constants k^i cannot both vanish for if they did then h would be closed and lemma 1 then gives $\Delta = 0$, contradicting $\Delta_0 > 0$.

Equation (2.23) determines B , which can be plugged into (2.19) to obtain

$$Z = \frac{\ell}{2} \left[-\frac{1}{3\Delta} \star_2 (\Gamma^{-1}k)' - \frac{1}{3} \Gamma^{-1}k + \frac{\Gamma'}{\Gamma} d\rho \right]. \quad (2.27)$$

Substituting the expression for B into (2.24) yields

$$k^i (\Gamma^{-1}k_i)' = \frac{3\Delta_0^2 \Gamma'}{\Gamma^4} \quad (2.28)$$

and, since k^i are constants, this can be integrated to give

$$k^i k_i = C^2 \Gamma - \frac{\Delta_0^2}{\Gamma^2} \quad (2.29)$$

where C is a positive constant. Now consider equation (2.8). The $\rho\rho$ component gives

$$h \cdot Z + \frac{3}{\ell} = \frac{\ell}{2} \left(\frac{\Gamma''}{\Gamma} - \frac{\Gamma'^2}{2\Gamma^2} \right). \quad (2.30)$$

The ij component gives

$$\frac{\ell\Gamma'}{4\Gamma} \gamma'_{ij} = -\frac{\Delta\ell\Gamma'}{4\Gamma} \sqrt{\gamma}\epsilon_{ij} + \gamma_{ij} (h \cdot Z + 3/\ell) - Z_i (h + (3/\ell)Z)_j. \quad (2.31)$$

Let's deal first with the special case in which Γ is constant. These two equations reveal that this implies $h = -(3/\ell)Z$ (Z_i cannot vanish when $\Gamma' = 0$ since this would imply that Z vanishes but Z has unit norm). All solutions with this property were obtained in [1]. There are three possibilities: corresponding to the metric on H being a homogeneous metric on the manifolds Nil , $SL(2, R)$ or S^3 respectively.

Now assume Γ is non-constant. Multiply (2.31) by γ^{ij} to get

$$\frac{\Gamma''}{\Gamma} = \frac{\Gamma'}{2\Gamma} (\log \gamma)'. \quad (2.32)$$

Since Γ' is nonzero, we can divide by Γ' and integrate to get

$$\sqrt{\gamma} = \beta^2 |\Gamma'|, \quad (2.33)$$

where β is a positive constant.

Multiplying (2.31) by $k^i k^j$ leads to an equation which can be rearranged to read

$$(k \cdot Z)^2 = C^2 \Gamma - \frac{\Delta_0^2}{\Gamma^2} - \frac{C^2 \ell^2 \Gamma'^2}{4\Gamma}. \quad (2.34)$$

But from equation (2.30) we have

$$k \cdot Z = \frac{\ell}{2} \Gamma'' + \frac{\ell\Gamma'^2}{4\Gamma} - \frac{3\Gamma}{\ell} = \frac{1}{\ell\Gamma} \frac{dy}{d\Gamma}, \quad (2.35)$$

where

$$y = \frac{\ell^2}{4} \Gamma \Gamma'^2 - \Gamma^3. \quad (2.36)$$

This implies that (2.34) can be rewritten as

$$\left(\frac{dy}{d\Gamma} \right)^2 + C^2 \ell^2 y = -\Delta_0^2 \ell^2, \quad (2.37)$$

with solution

$$y = -\frac{C^2 \ell^2}{4} (\Gamma - \alpha_0)^2 - \frac{\Delta_0^2}{C^2}, \quad (2.38)$$

where α_0 is a constant of integration. Hence we have

$$\Gamma'^2 = \frac{4P(\Gamma)}{\ell^2 \Gamma}, \quad (2.39)$$

where

$$P(\Gamma) = \Gamma^3 - \frac{C^2 \ell^2}{4} (\Gamma - \alpha_0)^2 - \frac{\Delta_0^2}{C^2}. \quad (2.40)$$

Note that $P(\Gamma) \geq 0$ implies that k cannot vanish anywhere unless $\alpha_0^3 = \Delta_0^2/C^2$.

At this point, it is convenient to exploit the $GL(2, R)$ symmetry to set $k^1 = 1$, $k^2 = 0$. Equation (2.29) then gives

$$\gamma_{11} = C^2 \Gamma - \frac{\Delta_0^2}{\Gamma^2}. \quad (2.41)$$

Plugging equation (2.39) into equation (2.35) and using the explicit form of Z (equation (2.27)) gives an ODE for γ_{12} :

$$\frac{d}{d\Gamma} \left(\frac{\gamma_{12}}{\gamma_{11}} \right) = \frac{\Delta_0 \beta^2}{(C^2 \Gamma^3 - \Delta_0^2)^2} (2C^2 \Gamma^3 - 3C^2 \alpha_0 \Gamma^2 + \Delta_0^2). \quad (2.42)$$

This can be integrated to give

$$\gamma_{12} = \frac{\Delta_0 \beta^2 (\alpha_0 - \Gamma)}{C^2 \Gamma^3 - \Delta_0^2} \gamma_{11}, \quad (2.43)$$

plus a constant times γ_{11} which can be eliminated using the remaining $GL(2, R)$ freedom to shift x^1 by a constant times x^2 . Finally we can get γ_{22} from (2.33), using the freedom to rescale x^2 to set $\beta = 1$. The 2-metric γ_{ij} is now fully determined in terms of Γ , so it is convenient to use Γ , instead of ρ as the 3rd coordinate on H . The full near-horizon solution is

$$g_{ab} dx^a dx^b = \frac{\ell^2 \Gamma d\Gamma^2}{4P(\Gamma)} + \left(C^2 \Gamma - \frac{\Delta_0^2}{\Gamma^2} \right) \left(dx^1 + \frac{\Delta_0 (\alpha_0 - \Gamma)}{C^2 \Gamma^3 - \Delta_0^2} dx^2 \right)^2 + \frac{4\Gamma P(\Gamma)}{\ell^2 (C^2 \Gamma^3 - \Delta_0^2)} (dx^2)^2, \\ \Delta = \frac{\Delta_0}{\Gamma^2}, \quad k = \frac{\partial}{\partial x^1}, \quad h = \Gamma^{-1} (k - d\Gamma), \quad (2.44)$$

where

$$P(\Gamma) = \Gamma^3 - \frac{C^2 \ell^2}{4} (\Gamma - \alpha_0)^2 - \frac{\Delta_0^2}{C^2} \quad (2.45)$$

with C and Δ_0 positive constants and α_0 an arbitrary constant. It can be checked that the remaining equations of section 2.2 are all satisfied.

To summarize, we have determined all non-static near-horizon solutions admitting two commuting Killing fields. There are several cases:

- I If Γ is constant then the near-horizon solution is one of the solutions given in section 3.2 of [1], for which H is locally isometric to a homogeneous $SL(2, R)$, Nil or S^3 manifold. The latter case arises as the near-horizon limit of the asymptotically AdS black hole solutions constructed in [1].
- II If Γ is non-constant then the near-horizon solution is (2.44).

2.5 Static near-horizon geometry

We now turn to the static case. This is analyzed in appendix A, where it is proved that the near-horizon solution must be one of the following:

- III If Γ is constant then the near-horizon solution is the solution given in section 3.2 of [1] for which H is locally isometric to $R \times H^2$. This is the near-horizon limit of the supersymmetric black “string” of [20].
- IV A solution (A.18) that can be obtained by taking the limit $\Delta_0 \rightarrow 0$ (with other constants and the coordinates fixed) of the non-static solution (2.44). (This amounts to simply setting $\Delta_0 = 0$ in the solution (2.44).)
- V A solution (A.11) that can be obtained from (2.44) by setting $\Delta_0^2 = C^2 \Gamma_0^3$, $x^1 = \hat{x}^2/C$, $x^2 = C \ell^2 \hat{x}^1 / 4 \Gamma_0^{3/2}$ and taking $C \rightarrow 0$ with Γ_0 , α_0 , Γ and \hat{x}^i fixed.
- VI The AdS_5 solution obtained in section 3.2 of [1], for which H is locally isometric to hyperbolic space.

The near-horizon geometry of the supersymmetric black holes of [3] is non-static with non-constant Γ and hence must be described by (II) which we shall prove below. We shall also see that solution (IV) describes the near-horizon geometry of a supersymmetric black ring but suffers from a conical singularity.

2.6 Global considerations

So far, the discussion has been local. However, we are interested mainly in solutions for which H is compact since this is true of any solution arising as the near-horizon limit of a black hole. A compact 3-manifold admitting a $U(1) \times U(1)$ symmetry must be homeomorphic to T^3 , $S^1 \times S^2$, S^3 or a lens space (see e.g. [21]). This excludes the solutions (I) and (III) with constant Γ except for the solution of [1] with H locally isometry to S^3 . The known black hole solutions with this near-horizon geometry [1] arise as a special case of those of [3], and since we shall show that the near-horizon limit of the latter solutions is described by (2.44), it follows that the S^3 solution with constant Γ can be obtained as a limit of (2.44), so we shall not consider this solution further.

The solution (VI) is excluded because H cannot be compactified without breaking the rotational symmetries. So consider the other solutions with non-constant Γ . Note that γ_{11} is a scalar invariant: it is the norm of $\partial/\partial x^1$ (which is some linear combination of m_1 and m_2). Furthermore, for all of the solutions with nonconstant Γ , γ_{11} is a monotonic function of Γ when $\Gamma > 0$. This implies that Γ can be expressed uniquely in terms of the invariant γ_{11} and is therefore globally defined.

If H is compact and Γ nonconstant then Γ must achieve a distinct minimum and maximum on H . Hence $d\Gamma$ must vanish at two distinct (positive) values of Γ . By calculating $(d\Gamma)^2$, one finds that this is not possible for solution (V) above so this solution is necessarily noncompact. This leaves solutions (II) and (IV). Hence any compact solution with nonconstant Γ must be described by the line element (2.44), whether static ($\Delta_0 = 0$) or not ($\Delta_0 > 0$).

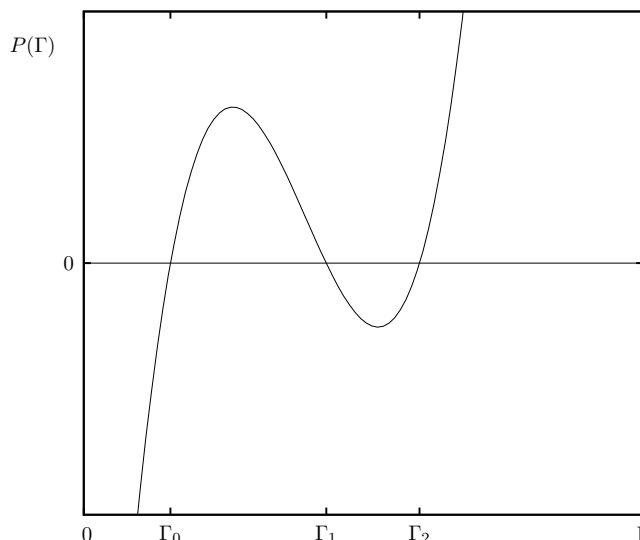


Figure 1: Sketch of $P(\Gamma)$ corresponding to a compact H . Note $P(\Gamma) > 0$ for $\Gamma_0 < \Gamma < \Gamma_1$.

For this solution we have $(d\Gamma)^2 = 4P(\Gamma)/(\ell^2\Gamma)$, so $P(\Gamma)$ must be non-negative. Compactness implies that $P(\Gamma)$ must have two distinct positive roots Γ_0, Γ_1 with $P(\Gamma)$ positive for $\Gamma_0 < \Gamma < \Gamma_1$. It is not hard to see that this is only possible if $P(\Gamma)$ has a third root $\Gamma_2 > \Gamma_1$. (We can't have $\Gamma_2 = \Gamma_1$ since this would make the proper distance to $\Gamma = \Gamma_1$ infinite, i.e., H would not be compact.) So, for a compact horizon, $P(\Gamma)$ must have the form sketched in figure 1. This imposes restrictions on the parameters of the solution.

At first sight, it appears that (2.44) is a 3-parameter solution. However, it is not hard to see that Γ is defined only up to multiplication by a positive constant. This freedom can be used to eliminate one of the parameters. More explicitly, if K is a positive constant then the (2.44) is invariant under the rescaling

$$\Gamma \rightarrow K\Gamma, \quad x^1 \rightarrow K^{-1}x^1, \quad C^2 \rightarrow KC^2, \quad \Delta_0 \rightarrow K^2\Delta_0, \quad \alpha_0 \rightarrow K\alpha_0. \quad (2.46)$$

2.7 $\Delta_0 = 0$: unbalanced black ring

The metric of H is

$$g_{ab}dx^a dx^b = \frac{\ell^2\Gamma d\Gamma^2}{4P(\Gamma)} + C^2\Gamma(dx^1)^2 + \frac{P(\Gamma)}{\Gamma^2}(dx^2)^2, \quad (2.47)$$

where we have rescaled x^2 . The metric has conical singularities at $\Gamma = \Gamma_0$ and $\Gamma = \Gamma_1$, where $\partial/\partial x^2$ vanishes. If one could remove these (by identifying x^2 with a suitable period) then one would be left with a smooth geometry of topology $S^1 \times S^2$ with the S^1 parameterized by x^1 and the S^2 by Γ and x^2 . (Note that in this case we have $m_i \propto \partial/\partial x^i$.) Thus it would describe the horizon of a supersymmetric black ring.

The necessary and sufficient condition for $P(\Gamma)$ to have distinct real positive roots is

$$0 < \frac{\alpha_0}{C^2\ell^2} < \frac{1}{27}. \quad (2.48)$$

It is convenient to define a parameter b by

$$\frac{\alpha_0}{C^2 \ell^2} = \frac{(1-b^2)^2}{(b^2+3)^3}, \quad (2.49)$$

where $0 < b < 1$. This defines b uniquely since the function on the r.h.s. decreases monotonically from $1/27$ at $b = 0$ to 0 at $b = 1$. We can now find explicit expressions for the roots:

$$\Gamma_0 = \frac{\alpha_0(b^2+3)}{4}, \quad \Gamma_1 = \frac{\alpha_0(b^2+3)}{(1+b)^2}, \quad \Gamma_2 = \frac{\alpha_0(b^2+3)}{(1-b)^2}. \quad (2.50)$$

The coordinate x^2 can be periodically identified to eliminate a conical singularity at $\Gamma = \Gamma_0$. The necessary and sufficient condition for the absence of a conical singularity of $\Gamma = \Gamma_1$ is then

$$\left(\frac{\Gamma_1}{\Gamma_0}\right)^{3/2} = \frac{\Gamma_2 - \Gamma_1}{\Gamma_2 - \Gamma_0}, \quad (2.51)$$

which implies $b = 1$. This is not allowed since $b < 1$ and hence one can never eliminate conical singularities from this metric.⁵ So, at best, this solution describes the near-horizon geometry of an *unbalanced* supersymmetric black ring: the conical singularity provides the force required to hold the ring in equilibrium.

Note that $b \rightarrow 1$ as $\ell \rightarrow \infty$, which suggests that the conical singularities *can* be eliminated as the cosmological constant is turned off. This is indeed true: in this limit, our $\Delta_0 = 0$ solution reduces to the $AdS_3 \times S^2$ near-horizon geometry of the supersymmetric black ring of [15].⁶

It is interesting to consider the five-dimensional near-horizon geometry, which, after a coordinate change $r = \Gamma R$, is

$$ds^2 = \Gamma \left[-C^2 R^2 dv^2 + 2dv dR + C^2 (dx^1 + R dv)^2 \right] + \frac{\ell^2 \Gamma d\Gamma^2}{4P(\Gamma)} + \frac{P(\Gamma)}{\Gamma^2} (dx^2)^2, \quad (2.52)$$

it can be checked that the expression in the square brackets is the line element of AdS_3 . Hence the five-dimensional near-horizon geometry is a warped product of AdS_3 and a deformed S^2 with a conical singularity at one pole.

Our solution can be oxidized on S^5 to give a solution of type IIB supergravity [22]. Viewing S^5 as an S^1 bundle over CP^2 , the ten-dimensional solution is a warped product of AdS_3 with a 7-manifold M_7 , which is an S^1 bundle over $S^2 \times CP^2$ where the S^2 is our singular S^2 . In fact, this ten-dimensional solution has been encountered before in a general exploration of such warped products [17]. Furthermore, it has been shown that the solution *can* be made into a globally regular metric if one takes M_7 to have a different topology, namely an S^1 bundle over a 6-manifold B_6 , where B_6 is an S^2 fibration over CP^2 [17]. The S^1 and S^2 here are not the same as for the uplift of our solution: if our Kaluza-Klein S^1 is

⁵In fact this is obvious since the l.h.s. of (2.51) is greater than one and the r.h.s. is less than one.

⁶To see this explicitly, define a new coordinate θ by $\Gamma = \Gamma_0 \cos^2 \theta + \Gamma_1 \sin^2 \theta$, use equation (2.49) to write ℓ in terms of C , α_0 and b and take the limit $b \rightarrow 1$ with C , α_0 and θ fixed and rescaling x^2 as appropriate. Note that Γ becomes constant in this limit.

parameterized by a coordinate ψ (and the S^2 by Γ and x^2) then the regular solution has S^1 parameterized by x^2 and S^2 parameterized by Γ and ψ .

So although our near-horizon geometry is necessarily singular in five dimensions, it can be made regular if lifted to ten dimensions. However, the resulting solution can no longer be reduced to five dimensions so one loses any interpretation in terms of five-dimensional black holes. It is natural to wonder whether the regular ten-dimensional solution might describe the near-horizon geometry of a supersymmetric, asymptotically $AdS_5 \times S^5$, black hole with horizon topology $S^1 \times M_7$.

2.8 $\Delta_0 > 0$: topologically spherical black hole

Write the metric on H as

$$g_{ab}dx^a dx^b = \frac{\ell^2 \Gamma d\Gamma^2}{4P(\Gamma)} + A(\Gamma) (dx^1 + \omega(\Gamma)dx^2)^2 + B(\Gamma)(dx^2)^2. \quad (2.53)$$

There are two cases to analyze depending on whether or not α_0 coincides with Γ_0 (note that α_0 cannot coincide with Γ_1 since $P'(\Gamma_1) < 0$ and $P'(\alpha_0) > 0$). If $\alpha_0 \neq \Gamma_0$ then $A(\Gamma) = (C^2/\Gamma^2)[P(\Gamma) + (C^2\ell^2/4)(\Gamma - \alpha_0)^2]$ is positive for $\Gamma_0 \leq \Gamma \leq \Gamma_1$. We shall treat this case here. The special case $\alpha_0 = \Gamma_0$ is treated separately in appendix B. It turns out that it can be obtained as a limit of the generic case with no further complications.

The range of Γ is $\Gamma_0 \leq \Gamma \leq \Gamma_1$. The 2-metric γ_{ij} is non-degenerate within this range but degenerates at the endpoints, where $B(\Gamma)$ vanishes. This implies that the Killing field $\omega(\Gamma_i)\partial/\partial x^1 - \partial/\partial x^2$ vanishes at $\Gamma = \Gamma_i$, $i = 0, 1$. Now, using the fact that Γ_i is a root of $P(\Gamma)$, we have

$$\omega(\Gamma_i) = \frac{4\Delta_0}{C^4\ell^2(\alpha_0 - \Gamma_i)}, \quad (2.54)$$

which implies that $\omega(\Gamma_0) \neq \omega(\Gamma_1)$. Hence the Killing field that vanishes at $\Gamma = \Gamma_0$ is distinct from the one that vanishes at $\Gamma = \Gamma_1$. In order to avoid conical singularities at $\Gamma = \Gamma_0, \Gamma_1$ we must assume that these two Killing fields have closed orbits, in other words they generate rotational symmetries. Hence our two rotational Killing fields m_i must be proportional to these two Killing fields:

$$m_1 = -d_1 \left(\omega(\Gamma_0) \frac{\partial}{\partial x^1} - \frac{\partial}{\partial x^2} \right), \quad m_2 = -d_2 \left(\omega(\Gamma_1) \frac{\partial}{\partial x^1} - \frac{\partial}{\partial x^2} \right), \quad (2.55)$$

for some non-zero constants d_1, d_2 . Introducing adapted coordinates ϕ_i so that $m_i = \partial/\partial\phi_i$ we have

$$x^1 = -[\omega(\Gamma_0)d_1\phi_1 + \omega(\Gamma_1)d_2\phi_2], \quad x^2 = d_1\phi_1 + d_2\phi_2. \quad (2.56)$$

The condition $\phi_i \sim \phi_i + 2\pi$ fixes the constants d_i up to signs: in order to avoid conical singularities one must take

$$|d_1| = \frac{\ell^2(C^2\Gamma_0^3 - \Delta_0^2)^{\frac{1}{2}}}{2P'(\Gamma_0)}, \quad |d_2| = \frac{\ell^2(C^2\Gamma_1^3 - \Delta_0^2)^{\frac{1}{2}}}{2|P'(\Gamma_1)|}. \quad (2.57)$$

The solution is now globally regular. It is clear that H has S^3 topology,⁷ with m_1 vanishing at $\Gamma = \Gamma_0$ and m_2 vanishing at $\Gamma = \Gamma_1$. In appendix B it is shown that the coordinate change from (x^1, x^2) to (ϕ_1, ϕ_2) is also valid for the special case $\alpha_0 = \Gamma_0$ (although $d_1 \rightarrow 0$ as $\alpha_0 \rightarrow \Gamma_0$, the product $d_1\omega(\Gamma_0)$ remains nonzero in this limit). Hence we do not need to treat this case separately any longer.

We shall now show that the near-horizon solution we have obtained is locally isometric to that of the supersymmetric black hole solution of Chong et al. [3]. It is convenient to use the roots Γ_i as parameters, as opposed to $(C^2, \alpha_0, \Delta_0)$. These are related by

$$C^2 = \frac{4}{\ell^2}(\Gamma_0 + \Gamma_1 + \Gamma_2) \quad \alpha_0 = \frac{\Gamma_0\Gamma_1 + \Gamma_0\Gamma_2 + \Gamma_1\Gamma_2}{2(\Gamma_0 + \Gamma_1 + \Gamma_2)}$$

$$\Delta_0^2 = C^2\Gamma_0\Gamma_1\Gamma_2 - \frac{C^4\ell^2\alpha_0^2}{4}. \tag{2.58}$$

Note that Γ_i are not totally arbitrary positive numbers: they are constrained by $\Delta_0^2 > 0$. Use the scale transformations (2.46) on Γ to fix

$$\frac{1}{\Gamma_0} + \frac{1}{\Gamma_1} - \frac{1}{\Gamma_2} = 2 \tag{2.59}$$

which can always be done (i.e. the corresponding $K > 0$) since $0 < \Gamma_0 < \Gamma_1 < \Gamma_2$. We can define two constants a, b so that the roots are parameterized as:

$$\Gamma_0 = \frac{1}{1+ag} \quad \Gamma_1 = \frac{1}{1+bg} \quad \Gamma_2 = \frac{1}{g(a+b)}, \tag{2.60}$$

where $g \equiv 1/\ell$. We need Γ_i to be positive and correctly ordered, which gives the restrictions

$$g^{-1} > a > |b|. \tag{2.61}$$

Note that b may be negative. One can solve for C , α_0 and Δ_0 using (2.58). One finds that

$$\Delta_0^2 = \frac{4(ag + bg + abg^2)}{(1+ag)^2(1+bg)^2(a+b)^2}, \tag{2.62}$$

so the constraint coming from $\Delta_0^2 > 0$ is, in the notation of [3],

$$r_0^2 \equiv g^{-1}(a+b) + ab > 0. \tag{2.63}$$

We now define a coordinate θ by

$$\Gamma = \frac{g\rho(\theta)^2}{(a+b)(1+ag)(1+bg)} \tag{2.64}$$

where

$$\rho(\theta)^2 = \frac{a+b}{g} + ab + a^2 \cos^2 \theta + b^2 \sin^2 \theta. \tag{2.65}$$

⁷More precisely, the covering space of H has S^3 topology. H could actually be a lens space if additional identifications are made.

It is easy to check that taking $0 \leq \theta \leq \pi/2$ does indeed uniquely parameterize the range $\Gamma_0 \leq \Gamma \leq \Gamma_1$ (although $\theta = 0$ corresponds to $\Gamma = \Gamma_1$ and $\theta = \pi/2$ to $\Gamma = \Gamma_0$). We then have

$$\cos^2 \theta = \frac{\Gamma - \Gamma_0}{g^2(a^2 - b^2)\Gamma_0\Gamma_1\Gamma_2}, \quad \sin^2 \theta = \frac{\Gamma_1 - \Gamma}{g^2(a^2 - b^2)\Gamma_0\Gamma_1\Gamma_2}, \quad (2.66)$$

$$P(\Gamma) = \frac{g(a-b)^2}{(1+ag)^3(1+bg)^3(a+b)} \cos^2 \theta \sin^2 \theta \Delta_\theta, \quad (2.67)$$

where

$$\Delta_\theta \equiv 1 - a^2 g^2 \cos^2 \theta - b^2 g^2 \sin^2 \theta, \quad (2.68)$$

and

$$\frac{\ell^2 \Gamma d\Gamma^2}{4P(\Gamma)} = \frac{\rho(\theta)^2 d\theta^2}{\Delta_\theta}. \quad (2.69)$$

For completeness, we give the expressions for the constants d_i :

$$d_1 = \frac{a + 2b + 2abg + gb^2}{g(1 - bg)(a - b)}, \quad d_2 = -\frac{b + 2a + 2abg + ga^2}{g(1 - ag)(a - b)} \quad (2.70)$$

We have checked that our 3-metric in the (θ, ϕ_1, ϕ_2) coordinates agrees exactly with the horizon metric of Chong et al. in their (θ, ψ, ϕ) coordinates⁸ with $\phi_1 = \psi$ and $\phi_2 = \phi$.

It is interesting to note that the near-horizon geometry is considerably simpler in the coordinates (Γ, x^1, x^2) than in the coordinates (θ, ϕ_1, ϕ_2) .

3. Discussion

In this paper we have determined the most general regular, supersymmetric, near-horizon geometry of an asymptotically AdS_5 black hole solution of minimal gauged supergravity which admits two rotational isometries. We found that the only such solution is the near-horizon geometry of the topologically spherical supersymmetric black hole discovered by Chong et al. [3]. Hence if new supersymmetric black hole solutions exist then either they do not have two rotational symmetries, or they have the same near-horizon geometry as the known solutions.

Our result implies that exotic topologies for supersymmetric black holes (such as black rings) are not allowed, unless they possess fewer than two rotational symmetries. We should emphasise that no known black hole in five dimensions possesses fewer than two rotational symmetries, including the asymptotically flat supersymmetric black rings of ungauged supergravity [15]. It has been conjectured, however, that there could exist asymptotically flat black holes with only one rotational symmetry [11], essentially because this is all one is guaranteed in general [16].

Curiously we did find a near-horizon geometry which describes a black ring, however it necessarily possesses a conical singularity. Physically this can be interpreted as suggesting

⁸Actually before taking the near horizon limit of their metric one needs to introduce coordinates which are valid on the horizon. In particular the angles ψ and ϕ must be shifted by functions of r (see the analysis in [5]) and it is these new angular variables which we are referring to.

that supersymmetric black rings in AdS cannot be "balanced", i.e., rotation and electromagnetic repulsion is not enough to counterbalance gravitational attraction (which is enhanced in AdS). However, known unbalanced ring solutions (e.g. [9, 10]) can be balanced by increasing the angular momentum, so it seems likely the same will be true here (the mass and/or charge would also have to change for consistency with the BPS inequality). Hence our singular near-horizon geometry may be interpreted as evidence in favour of the existence of regular non-supersymmetric anti-de Sitter black rings.

It is interesting to compare our result with the corresponding result for the ungauged theory [11]. In the latter theory, one obtains a complete classification assuming only supersymmetry and compactness of the horizon. The allowed near-horizon geometries all admit two rotational symmetries (in fact H is homogeneous), but this is an output, not an input. In the gauged theory, we had to input the rotational symmetries as an assumption. It would be nice if this assumption could be relaxed and the general solution of the equations of section 2.2 determined (for compact H).

In the ungauged theory it is possible to construct a global uniqueness argument for supersymmetric black holes of spherical topology [11]. It is obviously desirable to have a corresponding result for the gauged theory, i.e., prove that the solution of [3] is the only supersymmetric asymptotically AdS black hole solution, even assuming two rotational symmetries. However, the arguments of [11] rely heavily on features of the ungauged theory that do not extend to the gauged theory so new ideas would be required to do this.

In the ungauged theory, it is straightforward to extend the argument to allow for abelian vector multiplets [23] and the same is probably true in the gauged theory. So it seems likely that any supersymmetric, asymptotically AdS black hole solution of gauged supergravity coupled to abelian vector multiplets must have the same near-horizon geometry as the solution of [5] if it admits two rotational symmetries.

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A. Static near-horizon geometry: $\Delta_0 = 0$

The analysis can be divided into three subcases.

(I) First, consider $k \equiv 0$ and $B \equiv 0$. Then $h = -(2/\ell)Z$. It was shown in section 3.2 of [1] that this implies that the solution is just AdS_5 with vanishing Maxwell field. H is locally isometric to hyperbolic space, and hence cannot be compactified without breaking the rotational symmetries.

(II) Consider the case $k \equiv 0$ and B not identically zero. Equation (2.25) implies

$$\star_2 B = \Gamma^{-1} \omega, \tag{A.1}$$

where ω_i are constants (note $\omega_i = 0$ is covered by case (I) and thus we will assume this is not the case here). We have

$$h = -\frac{\Gamma'}{\Gamma}d\rho, \quad Z = \frac{\ell}{2} \left(\Gamma^{-1}\omega + \frac{\Gamma'}{\Gamma}d\rho \right). \quad (\text{A.2})$$

The $\rho\rho$ component of (2.8) gives

$$\frac{\Gamma''}{\Gamma} + \frac{\Gamma'^2}{2\Gamma^2} = \frac{6}{\ell^2}, \quad (\text{A.3})$$

which implies that Γ' is not identically zero hence we can change variable from ρ to Γ and rewrite this equation as

$$\frac{d}{d\Gamma} \left(\Gamma\Gamma'^2 - \frac{4\Gamma^3}{\ell^2} \right) = 0, \quad (\text{A.4})$$

with solution

$$\Gamma'^2 = \frac{4}{\ell^2\Gamma}P(\Gamma), \quad (\text{A.5})$$

where

$$P(\Gamma) = \Gamma^3 - \Gamma_0^3, \quad (\text{A.6})$$

and for convenience the integration constant has been chosen such that Γ_0 is the real root of $P(\Gamma)$. Using the fact that Z is a unit one-form one can deduce that:

$$\omega^i\omega_i = \frac{4\Gamma_0^3}{\ell^2\Gamma} \quad (\text{A.7})$$

and thus $\Gamma_0 > 0$. The ij component of (2.8) reduces to

$$\frac{\ell\Gamma'}{4\Gamma}\gamma_{ij}' = \frac{1}{\ell} \left(1 + \frac{2\Gamma_0^3}{\Gamma^3} \right) \gamma_{ij} - \frac{3\ell}{4\Gamma^2}\omega_i\omega_j. \quad (\text{A.8})$$

We can use the $GL(2, R)$ freedom to set $\omega_1 = 1$ and $\omega_2 = 0$, i.e. $\omega = dx^1$. Therefore γ_{12} and γ_{22} satisfy the same first order ODE which can be written as:

$$\frac{d}{d\Gamma} \log \gamma_{i2} = \frac{P'(\Gamma)}{P(\Gamma)} - \frac{2}{\Gamma}. \quad (\text{A.9})$$

Thus γ_{12} and γ_{22} are both equal to some constant times $P(\Gamma)/\Gamma^2$. This means that we can use the remaining $GL(2, R)$ freedom to set $\gamma_{12} = 0$ and $\gamma_{22} = P(\Gamma)/\Gamma^2$. Multiplying (A.8) by γ^{ij} implies

$$\frac{d}{d\Gamma} \log \gamma = \frac{P'(\Gamma)}{P(\Gamma)} - \frac{1}{\Gamma} \quad (\text{A.10})$$

where γ is the determinant of γ_{ij} , and hence $\gamma = C^2P(\Gamma)/\Gamma$ where C is a positive constant. These results imply $\gamma_{11} = C^2\Gamma$. Note that $\omega_i\omega^i = \gamma^{11}$ and thus upon comparison with (A.7) we deduce $C^2 = \ell^2/4\Gamma_0^3$. The rest of the equations in section 2.2

are satisfied without further constraint. Using Γ instead of ρ as a coordinate allows one to write the metric on the horizon as:

$$g_{ab}dx^a dx^b = \frac{\ell^2 \Gamma d\Gamma^2}{4P(\Gamma)} + \frac{\ell^2}{4\Gamma_0^3} \Gamma (dx^1)^2 + \frac{P(\Gamma)}{\Gamma^2} (dx^2)^2, \quad (\text{A.11})$$

This metric is smooth for $\Gamma > \Gamma_0$ (recall necessarily we have $\Gamma_0 > 0$). There is a conical singularity at $\Gamma = \Gamma_0$ but this can be removed by appropriately identifying x^2 . The 3-manifold this defines is topologically $R^2 \times S^1$. By rescaling x^1 suitably and letting $\Gamma_0 \rightarrow 0$ we recover case (I).

(III) Consider the case where k is not identically zero. Equation (2.23) implies that $\Gamma^{-1}k_i = \text{const}$, and therefore one can again deduce from (2.25) that:

$$\star_2 B = \Gamma^{-1} \omega \quad (\text{A.12})$$

where ω_i are constants. Expanding $\star_2 B$ in the basis $\{k, \star_2 k\}$ and using (2.24) implies

$$\omega = \alpha_0 \Gamma^{-1} k \quad (\text{A.13})$$

for some constant α_0 . Therefore

$$Z = \frac{\ell}{2} \left[\left(\frac{\alpha_0}{\Gamma} - 1 \right) \Gamma^{-1} k + \frac{\Gamma'}{\Gamma} d\rho \right]. \quad (\text{A.14})$$

Note that if Γ is a constant, $h = \alpha Z$ for some constant α . This case was considered in section 3.2 of [1], where it was proved that H must be locally isometric to $R \times H^2$. In the following we assume Γ is non-constant. From the fact that Z is a unit one-form one can show that:

$$\Gamma'^2 = \frac{4P(\Gamma)}{\ell^2 \Gamma} \quad (\text{A.15})$$

where

$$P(\Gamma) = \Gamma^3 - \frac{C^2 \ell^2}{4} (\Gamma - \alpha_0)^2 \quad (\text{A.16})$$

and we have defined the constant $C > 0$ by $C^2 = \Gamma^{-1} k_i k^i$ (since k^i are constants by assumption). Let us use the $GL(2, R)$ freedom available to set $k^1 = 1$ and $k^2 = 0$. Using the definition of the constant C then gives $\gamma_{11} = C^2 \Gamma$. Also, since $\Gamma^{-1} k_i$ is a constant we deduce that $\gamma_{12} = c\Gamma$ for some constant c . Using the $GL(2, R)$ transformations (which leave k^i invariant), we can arrange to have $\gamma_{12} = 0$. Then, the 22 component of equation (2.8) simplifies to:

$$\frac{d}{d\Gamma} \log \gamma_{22} = \frac{P'(\Gamma)}{P(\Gamma)} - \frac{2}{\Gamma} \quad (\text{A.17})$$

which implies $\gamma_{22} = P(\Gamma) \Gamma^{-2}$ (the integration constant have been fixed using the remaining $GL(2, R)$ freedom). Collecting these results gives the horizon geometry

$$g_{ab}dx^a dx^b = \frac{\ell^2 \Gamma d\Gamma^2}{4P(\Gamma)} + C^2 \Gamma (dx^1)^2 + \frac{P(\Gamma)}{\Gamma^2} (dx^2)^2 \quad (\text{A.18})$$

with $P(\Gamma)$ given by (A.16). The rest of the equations in section 2.2 are now satisfied without further constraint. This metric is analysed in section 2.7 of the main text.

B. Special case $\alpha_0 = \Gamma_0$

This special case needs to be considered separately. The metric on H still takes the form (2.53). For $\Gamma_0 \leq \Gamma \leq \Gamma_1$ the function $A(\Gamma)$ now vanishes for $\Gamma = \Gamma_0$ and is positive otherwise. The function $B(\Gamma)$ now vanishes only at one end point $\Gamma = \Gamma_1$ and is positive in the rest of the interval. Therefore the 2-metric γ_{ij} is non-degenerate for $\Gamma_0 < \Gamma < \Gamma_1$ and degenerates at $\Gamma = \Gamma_0$ and $\Gamma = \Gamma_1$. The Killing field $\partial/\partial x^1$ vanishes at $\Gamma = \Gamma_0$ and the Killing field $\omega(\Gamma_1)\partial/\partial x^1 - \partial/\partial x^2$ vanishes at $\Gamma = \Gamma_1$. In order to avoid conical singularities at $\Gamma = \Gamma_0, \Gamma_1$ these two Killing fields must have closed orbits. Thus these two Killing fields must be proportional to m_i , say

$$m_1 = c_1 \frac{\partial}{\partial x^1}, \quad m_2 = -d_2 \left(\omega(\Gamma_1) \frac{\partial}{\partial x^1} - \frac{\partial}{\partial x^2} \right). \quad (\text{B.1})$$

Now introduce adapted coordinates such that $m_i = \partial/\partial\phi_i$:

$$x^1 = c_1\phi_1 - d_2\omega(\Gamma_1)\phi_2, \quad x^2 = d_2\phi_2. \quad (\text{B.2})$$

The condition $\phi_i \sim \phi_i + 2\pi$ implies that in order to avoid the conical singularities we must take:

$$c_1^2 = \frac{\ell^2}{9C^2\Gamma_0}, \quad d_2^2 = \frac{C^4\ell^6(\Gamma_1 - \Gamma_0)^2}{16P'(\Gamma_1)^2}. \quad (\text{B.3})$$

This solution is now globally regular and has S^3 topology with m_1 vanishing at $\Gamma = \Gamma_0$ and m_2 vanishing at $\Gamma = \Gamma_1$. The coordinate change $(x^1, x^2) \rightarrow (\phi_1, \phi_2)$ can be obtained from the $\alpha_0 \neq \Gamma_0$ case studied in section 2.8 by taking the limit $\alpha_0 \rightarrow \Gamma_0$.

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